

Ch.5 Trigonometric Functions of Angles

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November 2020

1 5.1 Circles

p.343

1. Given: Find the distance between (5,3) and (-1,-5)

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(5 - (-1))^2 + (3 - (-5))^2}$$

$$\sqrt{6^2 + 8^2}$$

$$\sqrt{36 + 64}$$

$$\sqrt{100}$$

$$\text{distance} = 10$$

It was pretty easy to do out the problem but I was a little bit confused on which was x/y_1 and x/y_2 There was nothing that I did not understand in this problem.

3. center=(8,-10) radius=8

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 8)^2 + (y + 10)^2 = 8^2$$

$$(x - 8)^2 + (y + 10)^2 = 64$$

This problem was a little bit difficult to wrap my head around in the beginning and the h and k was one of those things that I could never remember which is which.

5. circle that goes through (7,-2) and (-10,0)

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 7)^2 + (y - (-2))^2 = r^2$$

$$((-10) - 7)^2 + (0 - (-2))^2 = r^2$$

$$\begin{aligned}
-(10 + 7)^2 + (0 + 2)^2 &= r^2 \\
-17^2 + 2^2 &= r^2 \\
289 + 4 &= r^2 \\
r^2 &= 293
\end{aligned}$$

$$(x - 7)^2 + (y + 2)^2 = 293$$

This problem was kind of confusing. There was kind of a lot to deal with in front of me. The concept of the equation was easy but I was a bit confused as to which points go in what places.

7. Write an equation for a circle with (2,6) and (8,10) which are on the diameter

$$\begin{aligned}
\text{Distance} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
(x - h)^2 + (y - k)^2 &= r^2
\end{aligned}$$

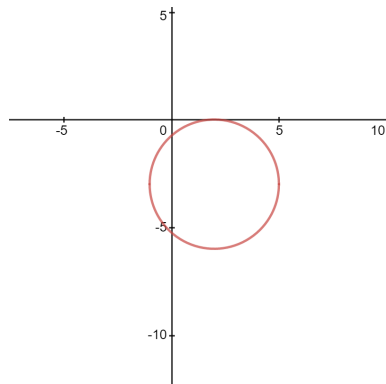
$$\begin{aligned}
&\sqrt{(8 - 2)^2 + (10 - 6)^2} \\
&\sqrt{6^2 + 4^2} \\
&\sqrt{36 + 16} \\
&\sqrt{52} \\
&2\sqrt{13}
\end{aligned}$$

$$\begin{aligned}
r &= \sqrt{13} \text{ so } r^2 = 13 \\
h &= (8+2)/2 \quad k = (10+6)/2 \\
h &= 10/2 \quad k = 16/2 \\
h &= 5 \quad k = 8
\end{aligned}$$

$$(x - 5)^2 + (y - 8)^2 = 13$$

This one was a bit complex but the equation was easy to follow. It takes a while to make sure that it is formatted well but over all it was very doable

9. $(x - 2)^2 + (y + 3)^2 = 9$



This problem is very easy. The graph was extremely simple and quick to complete. There were no parts of this problem that I found very difficult.

11. find the y-intercept center=(2,3) radius=3

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(0 - 2)^2 + (y - 3)^2 = 3^2$$

$$-2^2 + (y - 3)^2 = 3^2$$

$$4 + (y - 3)^2 = 9$$

$$(y - 3)^2 = 5$$

$$y - 3 = \pm\sqrt{5}$$

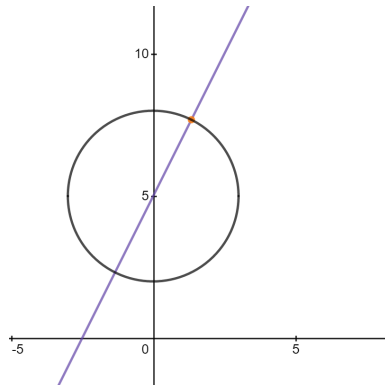
$$y = 3 + \pm\sqrt{5}$$

$$(0, 3 + \sqrt{5}), (0, 3 - \sqrt{5})$$

This problem was very confusing and I had to look at the solution manual for help. I had no idea how I was supposed to do this. I tried to find help from a few sources none of which explained very well. To understand this better I will use Khan Academy.

13. With the line $y=2x+5$ and the circle centered at $(0,5)$ with a radius of 3 find where they intersect in the first quadrant

(1.342, 7.683)

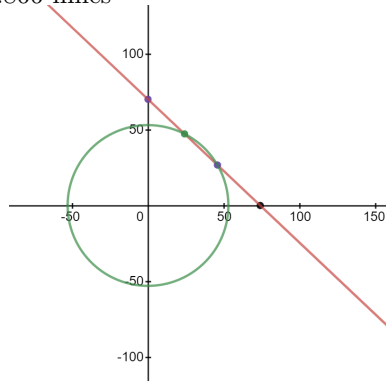


This problem was easy to complete. I just had to plug the information given into Desmos and it gave me exactly what I needed.

17. If the radius of service was 53 miles (centered at the origin), for the distance between, 70 miles north/ $(0,70)$ and 74 miles east/ $(74,0)$ how much of it would be in the circle

$$\begin{aligned} \text{Distance} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(45.794 - 24.098)^2 + (26.681 - 47.205)^2} \\ &= \sqrt{21.696^2 + (-20.524)^2} \\ &= \sqrt{470.716416 + 421.234576} \\ &= \sqrt{891.950992} \end{aligned}$$

The distance between the two towns that is within the circle is approximately 29.866 miles



I was a bit unsure as to which point is point one and which is point two. The easy part was getting most of the data because plugging the information given into desmos makes it easier.

2 5.2 Angles

p.359

5. Change $(5\pi)/6$ from radians to degrees

$$\begin{aligned} &(5\pi)/6 * 180/\pi \quad \pi \text{ cancels out} \\ &5/6 * 180 \\ &5/6 * ((6)30) \quad \text{The 6 cancels out} \\ &5 * 30 \end{aligned}$$

150 degrees

This question was a bit difficult at first because this concept is completely new to me. However the idea of it is really easy and the equation was easy to solve.

11. finding the coterminal angle of $26\pi/9$

(angle- 2π)

$$\begin{aligned}(26\pi/9) - 2\pi \\ 2\pi/1 * 9/9 = 18\pi/9 \\ 26\pi/9 - 18\pi/9 = 8\pi\end{aligned}$$

$$8\pi/9$$

Coterminal angles were a bit confusing but once I found the equation that I was supposed to use it made a lot more sense and I could finish it quickly.

.360

17. On a circle of radius 12cm find the length of the arc that subtends a central angle of 120 degrees

$$s=r\theta$$

$$\begin{aligned}120 * \pi/180 \\ (60 * 2) * \pi / (60 * 3) \text{ The 60s cancel out} \\ 2\pi/3 \\ 12 * 2\pi/3 \\ (3 * 4) * 2\pi/3 \text{ The 3s cancel out}\end{aligned}$$

$$8\pi \text{ cm}$$

At first this looked kind of intimidating but then I realized that it is almost like factoring for the most part it was a lot easier than it first seemed.

25. A truck with 32-in.-diameter wheels is traveling at 60 mi/h. Find the angular speed of the wheels in rad/min. How many revolutions per minute do the wheels make?

$$v=rw$$

$$\begin{aligned}r=16\text{in } v=60\text{mi/h}=1\text{mi/min}=63360\text{in/min} \\ w=v/r=63360/16=3960\text{rad/min} \\ 3960/2\pi \\ (2 * 1980)/2\pi \text{ 2s cancel out} \\ 1980/\pi = 630.25357464\end{aligned}$$

$$630.25\text{rev/min}$$

This one was very confusing. I had to use the solution manual and I am not really sure that I completely understand how to do it. To try and understand,

I will do some khan academy exercises.

26. A bicycle with 24-in.-diameter wheels is traveling at 15 mi/h. Find the angular speed of the wheels in rad/min. How many revolutions per minute do the wheels make?

$$x * \Pi / 180$$
$$w = v / r$$

$$r = 12 \text{ in } v = 15 \text{ mi/h} = 0.5 \text{ mi/min} = 31680 \text{ in/min}$$
$$w = v / r = 31680 / 12 = 2640 \text{ rad/min}$$
$$2640 / 2\Pi$$
$$(2 * 1320) / 2\Pi \text{ 2s cancel out}$$
$$1320 / \Pi = 420.16904976$$

$$420.17 \text{ rev/min}$$

This question was about as difficult as the last question on this topic. I do not really understand angular speed and to fix that I will go to khan academy. Something just hasn't clicked.

3 5.3 Points on Circles Using Sine and Cosine

p.373

1. Find the quadrant in which the terminal point determined by t lies if a.
 $\sin(t) < 0$ and $\cos(t) < 0$
b. $\sin(t) > 0$ and $\cos(t) < 0$

a. $\sin(t) < 0$ and $\cos(t) < 0$
 \sin and \cos are both only negative in quadrant 3

(t) is in quadrant 3

These were very simple to find out. I just had to use that trigonometry circle chart,

3. The point P is on the unit circle. If the y-coordinate of P is $3/5$, and P is in quadrant II, find the x coordinate.

$$\cos^2 + \sin^2 = 1$$

$$\cos^2 + (3/5)^2 = 1$$
$$\cos^2 + 9/25 = 1$$

$$\cos^2 = 16/25$$

$$\sqrt{16/25} = 4/5$$

$$x=4$$

This topic to me was confusing. I can't really follow the steps for solving the equation. I don't know how you get from one to the next. I will do khan academy practice to figure this out.

6. If $\cos(\theta)=2/9$ and θ is in the 1st quadrant, find $\sin(\theta)$.

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

$$\cos(2/9)^2 + \sin^2(\theta) = 1$$

$$\cos(4/81) + \sin^2(\theta) = 1$$

$$\sin^2(\theta) = 77/81$$

$$\sqrt{77/81}$$

$$\sqrt{77}/9$$

This problem was fairly easy to understand. I did this problem without any help except for how to format the answer. It was a bit unclear if I was supposed to leave it or not so I had to check the solution manual for that.

11. For each of the following angles, find the reference angle and which quadrant the angle lies in. Then compute sine and cosine of the angle.

a. $5\pi/4$

$$5\pi/4 * 180/\pi \quad \pi \text{ cancels out}$$

$$5/4 * (4 * 45) \text{ the 4s cancel out}$$

$$5 * 45 = 225$$

$$45 * \pi/180$$

$$45 * \pi / (4 * 45) \text{ The 45s cancel}$$

$$\pi/4$$

$$-\sin(\pi/4) = -\sqrt{2}/2$$

reference angle = $\pi/4$ It is in quadrant 3 and $-\cos(\pi/4) = -\sqrt{2}/2$ and $-\sin(\pi/4) = -\sqrt{2}/2$

b. $7\pi/6$

$$7\pi/6 * 180/\pi \quad \pi \text{ cancels out}$$

$$7/6 * (6(30)) \text{ The 6s cancel out}$$

$$7 * 30 = 210$$

$$210 - 180 = 30$$

$$30 * \pi/180$$

$30 * \Pi / (6(30))$ The 30s cancel out

the reference angle is $\Pi/6$ and it is in quadrant 3 and $-\sin(30) = -1/2$ $-\cos(30) = -\sqrt{3}/2$

c. $5\Pi/3$
 $5\Pi/3 * 180/\Pi$ Π cancels out
 $5/3 * (3(60))$
 $5 * 60 = 300$
 $360 - 300 = 60$
 $60 * \Pi / 180$
 $60 * \Pi / (3(60))$

reference angle = $\Pi/3$ it is in quadrant 4 and $\cos(60) = 1/2$ and $-\sin(60) = -\sqrt{3}/2$

d. $3\Pi/4$
 $3\Pi/4 * 180/\Pi$ Π cancels out
 $3/4 * 180$
 $3/4 * (4(45))$ the 4s cancel out
 $3 * 45 = 135$
 $180 - 135 = 45$
 $45 * \Pi / 180$
 $45 * \Pi / (4 * 45)$ The 45s cancel

reference angle = $\Pi/4$ this is in quadrant 2 and $-\cos(\Pi/4) = -\sqrt{2}/2$ and $\sin(\Pi/4) = \sqrt{2}/2$

This question took a long time to complete but the math was pretty easy for the most part. The steps to the solution are very easy to understand.

p. 374

13. Give exact values for $\sin()$ and $\cos()$ for each of these angles.

a. $-3\Pi/4$
 $-3\Pi/4 + 2\Pi$
 $-3\Pi/4 + 8\Pi/4 = 5\Pi/4$
 $5\Pi/4 * 180/\Pi$ Π cancels out
 $5/4 * (4(45))$ The 4s cancel out
 $5 * 45 = 225$
 $225 - 180 = 45$
 $45 * \Pi / 180$
 $45 * \Pi / (4(45))$ The 45s cancel out

reference angle = $\Pi/4$ quadrant 3 and $-\cos(\Pi/4) = -\sqrt{2}/2$ and $-\sin(\Pi/4) = -\sqrt{2}/2$

b. $23\pi/6$
 $23\pi/6 + 2\pi$
 $23\pi/6 - 12\pi/6 = 11\pi/6$
 $11\pi/6 * 180/\pi$ π cancels out
 $11/6 * (6(30))$ The 6s cancel out
 $11 * 30 = 330$
 $360 - 330 = 30$
 $30 * \pi/180$
 $30 * \pi/(6(30))$ The 30s cancel out

reference angle = $\pi/6$ quadrant 4 and $\cos(\pi/6) = \sqrt{3}/2$ and $-\sin(\pi/6) = -1/2$

c. $-\pi/2$
 $y = -1$ $x = 0$

$\cos(-\pi/2) = y/1 = -1$ and $\sin(-\pi/2) = x/1 = 0$

d. 5π
 $5\pi = 4\pi + \pi$

$\sin(5\pi) = \sin(\pi) = 0$ and $\cos(5\pi) = \cos(\pi) = -1$

At first the problem was kind of confusing and then as I did the parts it started to make more and more sense. I just needed the practice of those few parts for me to get it.

4 5.4 The Other Trigonometric Functions

p.382

1. $\theta = \pi/4$ $\sec(\pi/4) = 1/\cos(\pi/4) = 1/(\sqrt{2}/2) = 2/\sqrt{2} = \sqrt{2}$
 $\csc(\pi/4) = 1/(\sin(\pi/4)) = 1/(\sqrt{2}/2) = 2/\sqrt{2} = \sqrt{2}$
 $\tan(\pi/4) = (\sin(\pi/4)/\cos(\pi/4)) = (\sqrt{2}/2)/(\sqrt{2}/2) = 1$
 $\cot(\pi/4) = 1/\tan(\pi/4) = 1$

Finding the different parts is confusing so I looked in the solution manual and that made it a lot easier to understand, seeing it done out.

9. θ is in Quadrant 2
 $\cos(\theta) < 0$, $\sec(\theta) = 1/\cos(\theta)$
 $\sin(\theta) > 0$
 $\csc(\theta) = 1/\sin(\theta) > 0$
 $\tan(\theta) = \sin(\theta)/\cos(\theta) < 0$

$$\cot(\theta) = 1/\tan(\theta) < 0$$

This one was very difficult and I really don't understand much of anything about this problem. I could not do another one if I tried. I need to do some khan academy.

p.383

$$19. \sec(t)/\csc(t) \text{ simplify}$$

$$(1/\cos(t))/(1/\sin(t)) = \sin(t)/\cos(t)$$

$$\sec(t)/\csc(t) = \tan(t)$$

This problem is a bit confusing to figure it out, I had to look at the solution manual and I don't completely understand so I will use khan academy.

p.384

$$27. \sin^2(\theta)/1 + \cos(\theta) = 1 - \cos^2(\theta)$$

$$\sin^2(\theta)/1 + \cos(\theta) = 1 - \cos^2(\theta)/1 + \cos(\theta)$$

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$(1 + \cos(\theta))(1 - \cos(\theta))/1 + \cos(\theta)$$

$$1 - \cos(\theta)$$

None of this made sense. As I was going through it, I got more and more confused.

5 5.5 Right Triangle Trigonometry

p.391

1. find $\sin(A)$, $\cos(A)$, $\tan(A)$, $\sec(A)$, $\csc(A)$, $\cot(A)$. a and b=10 and 8

$$10^2 + 8^2 = c^2$$

$$100 + 64 = 164$$

$$\sqrt{164} = 2\sqrt{41}$$

$$\sin(A) = \text{opposite/hypotenuse} = 10/2\sqrt{41} = 5/\sqrt{41}$$

$$\cos(A) = \text{adjacent/hypotenuse} = 8/2\sqrt{41} = 4/\sqrt{41}$$

$$\tan(A) = \sin(A)/\cos(A) = (5/\sqrt{41})/(4/\sqrt{41}) = 5/4$$

$$\sec(A) = 1/\cos(A) = 1/(4/\sqrt{41}) = \sqrt{41}/4$$

$$\csc(A) = 1/\sin(A) = 1/(5/\sqrt{41}) = \sqrt{41}/5$$

$$\cot(A) = 1/\tan(A) = 1/(5/4) = 4/5$$

$$\sin(A) = 5/\sqrt{41} \cos(A) = 4/\sqrt{41} \tan(A) = 5/4 \sec(A) = \sqrt{41}/4 \csc(A) = \sqrt{41}/5 \cot(A) = 4/5$$

Parts of this made sense. Some of the steps made sense and some didn't. I will use khan academy.

3. solve for unknown sides and angles
sides= adjacent is 7 angles=90,30

$$\begin{aligned}\sin(30^\circ) &= 7/c \\ c &= 7/\sin(30) = 7/(1/2) = 14 \\ \tan(30 \text{ degrees}) &= 7/b \\ b &= 7/\tan(30) = 7/(1/\sqrt{3}) = 7\sqrt{3} \\ B &= 90 - 30 = 60 \text{degrees}\end{aligned}$$

$$c=14 \quad b=7\sqrt{3} \quad B=60^\circ$$

This was kind of confusing and overwhelming at first. The more I went through it the easier it got. It was just in time.

9. A 33-ft ladder leans against a building so that the angle between the ground and the ladder is 80° . How high does the ladder reach up the side of the building?

$$\begin{aligned}\sin(80^\circ) &= x/33 \\ \sin(80) &\approx 32.5\end{aligned}$$

The ladder reaches about 32.5ft. up the building

I was confused about this one so I had to use the solution manual. I will use khan academy.

p.392

19. find x in the triangle with a height of 82, and angles of 63 and 39

$$\begin{aligned}\tan(63^\circ) &= 82/a = a \\ a &= 82/\tan(63^\circ) \\ \tan(39^\circ) &= 82/b = b \\ b &= 82/\tan(39^\circ) \\ x &= a+b = (82/\tan(63^\circ)) + (82/\tan(39^\circ)) \approx 143.0427\end{aligned}$$

$$x \approx 143.043$$

This was kind of easy to follow especially because of the similar problems beforehand. I needed a little bit of

23. A plane is flying 2000 feet above sea level toward a mountain. The pilot observes the top of the mountain to be 18° above the horizontal, then immediately flies the plane at an angle of 20° above horizontal. The airspeed of the

plane is 100 mph. After 5 minutes, the plane is directly above the top of the mountain. How high is the plane above the top of the mountain (when it passes over)? What is the height of the mountain? [UW]

$$\begin{aligned}PT &= (110 \text{ mi} / 1 \text{ hr})(1 \text{ hr} / 60 \text{ min})(5 \text{ min}) = 25/3 \text{ mi} = 44000 \text{ ft.} \text{ in triangle } PTL, \sin(20^\circ) = TL/PT \\TL &= PT \sin(20^\circ) = 44000 \sin(20^\circ) \text{ ft} \\ \cos(20^\circ) &= PL/PT \\ PL &= PT \cos(20^\circ) = 44000 \cos(20^\circ) \text{ ft.}\end{aligned}$$

This was very confusing to me and I didn't really understand much at all and had to heavily use the solution manual.